LECTURE SERIES-05

DATE 22-04-20 | B. Sc(H)-II | * PHYSICS * | By. Dr. M.K. THAKUR
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Fresnell's Laws or Formulae: The amplitude of reflected and transmitted electromagnetic waves with that of incident one when the boundary is between two dielectric are called Fresnel's formulae. These contained in the boundary conditions,

$$(D_i)_n + (D_R)_n = (D_T)_n \quad \text{———(1)}$$
$$(B_i)_n + (B_R)_n = (B_T)_n \quad \text{———(2)}$$
$$(E_i)_n + (E_R)_n = (E_T)_n \quad \text{———(3)}$$
and $$(H_i)_t + (H_R)_t = (H_T)_t \quad \text{———(4)}$$

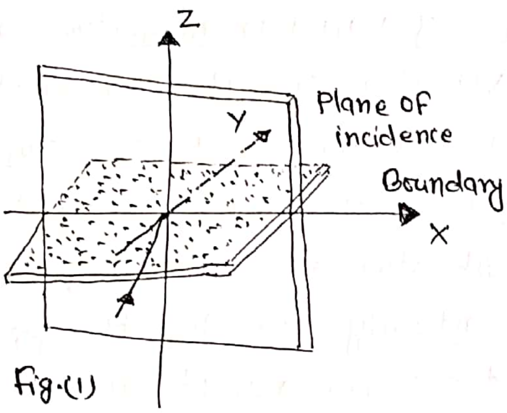

Fig.(1)

The condition(1) and (2) when coupled with Snell's Law yield no information and not included in the condition(2) and (4). So it is necessary to consider only condition (3) and (4)

Consider a plane E.M wave in x-y plane (plane of incidence) incident on a plane boundary (here x-y plane) and consider it as superposition of two waves one with the electric vector parallel to the plane of incidence and the other with electric vector perpendicular to the plane of incidence. Therefore it is sufficient to consider these two cases separately. The general result may be obtained from the appropriate linear combination of the two cases.

Case(I) $\vec{E}$ parallel to the plane of incidence.

In fig.(2). The electric and propagation vectors in two media are indicated. The direction of H vector are chosen so as to give a positive flow of energy in the direction of wave vector. In this situation the magnetic

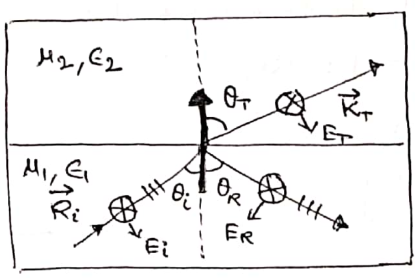

Fig.(2)

vectors are all parallel to the boundary surface.

$$\left.\begin{aligned}(H_i)_t &= H_i\\ (H_R)_t &= H_R\\ (H_T)_t &= H_T\end{aligned}\right] \quad \text{and} \quad \left.\begin{aligned}(E_i)_t &= E_i\cos\theta_i\\ (E_R)_t &= -E_R\cos\theta_R\\ (E_T)_t &= E_T\cos\theta_T\end{aligned}\right]$$

So boundary condition (3) and (4) reduce to

$$E_i\cos\theta_i - E_r\cos\theta_r = E_T\cos\theta_T \quad \text{—— (5)}$$

and

$$H_i + H_R = H_T \quad \text{—— (6)}$$

In equation (5) and (6) we have omitted the zero subscripts on E and H, it being understood that the phases now cancel and equ^n are relations between amplitudes.

Now as $\theta_i = \theta_r$ and $H = (E/z) = (\eta/\mu_r z_0)E$ $\left\{\because z = \frac{\mu_r}{\eta} z_0\right\}$

i.e $H = (\eta/z_0)E$ ($\because \mu_r = 1$, For dielectrics)

So equ^n (5) and (6) reduce to

$$E_i\cos\theta_i - E_R\cos\theta_R = E_T\cos\theta_T \quad \text{—— (7)}$$

and

$$\eta_1 E_i + \eta_1 E_R = \eta_2 E_T \quad \text{—— (8)}$$

The interest lies in the fraction of the incident amplitudes which are reflected and transmitted.

So eliminating E_T from equ^n (7) with the help of (8), we get.

$$(E_i - E_R)\cos\theta_i = \frac{\eta_1}{\eta_2}(E_i + E_R)\cos\theta_T$$

i.e $$\left(\frac{E_R}{E_i}\right)_{\parallel} = \frac{\frac{\eta_2}{\eta_1}\cos\theta_i - \cos\theta_T}{\frac{\eta_2}{\eta_1}\cos\theta_i + \cos\theta_T} \quad \text{—— (A')}$$

or $$\left(\frac{E_R}{E_i}\right)_{\parallel} = \frac{\left(\frac{\sin\theta_i}{\sin\theta_T}\right)\cos\theta_i - \cos\theta_T}{\left(\frac{\sin\theta_i}{\sin\theta_T}\right)\cos\theta_i - \cos\theta_T} \quad (\because \eta_1\sin\theta_i = \eta_2\sin\theta_T)$$

i.e $$\left(\frac{E_R}{E_i}\right)_{\parallel} = \frac{\sin\theta_i\cos\theta_i - \sin\theta_T\cos\theta_T}{\sin\theta_i\cos\theta_i + \sin\theta_T\cos\theta_T} = \frac{\sin 2\theta_i - \sin 2\theta_T}{\sin 2\theta_i + \sin 2\theta_T}$$

i.e $$\left(\frac{E_R}{E_i}\right)_{\parallel} = \frac{\tan(\theta_i - \theta_T)}{\tan(\theta_i + \theta_T)} \quad \text{—— (A)}$$

Teacher's Signature:

Similarly eliminating E_R from eqn (7) with help of eqn (8), we get

$$E_i \cos\theta_i - \left(\frac{n_2}{n_1} E_T - E_i\right)\cos\theta_i = E_T \cos\theta_T$$

i.e $$\left(\frac{E_T}{E_i}\right)_{\parallel} = \frac{2\cos\theta_i}{\left(\frac{n_2}{n_1}\cos\theta_i + \cos\theta_T\right)} \quad \text{———} \quad (B')$$

or $$\left(\frac{E_T}{E_i}\right)_{\parallel} = \frac{2\cos\theta_i}{\left(\frac{\sin\theta_i}{\sin\theta_T}\cos\theta_i + \cos\theta_T\right)}$$

or $$\left(\frac{E_T}{E_i}\right)_{\parallel} = \frac{2\cos\theta_i \sin\theta_T}{\sin\theta_i\cos\theta_i + \sin\theta_T\cos\theta_T}$$

i.e $$\left(\frac{E_T}{E_i}\right)_{\parallel} = \frac{2\cos\theta_i \sin\theta_T}{\sin(\theta_i+\theta_T)\cos(\theta_i-\theta_T)} \quad \text{———} \quad (B)$$

Case(II) $\vec{E}$ - Perpendicular to the plane of incidence.

In fig.(2) The magnetic field vector and the propagation vectors are indicated. The electric vector are directed into the plane of all directed into the plane of the figure.

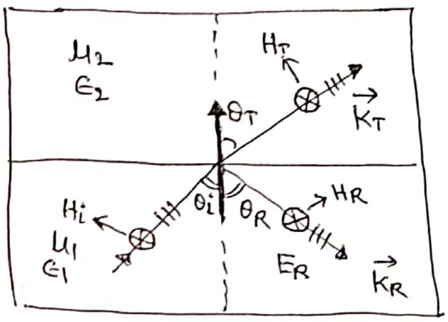

Fig.(3)

Since the electric vectors are parallel to the boundary surface.

$$\left.\begin{aligned}(E_i)_t &= E_i\\ (E_R)_t &= E_R\\ (E_T)_t &= E_T\end{aligned}\right] \text{ and } \begin{aligned}(H_i)_t &= -H_i\cos\theta_i\\ (H_R)_t &= H_R\cos\theta_R\\ (H_T)_t &= -H_T\cos\theta_T\end{aligned}$$

So boundary conditions (3) and (4) reduces to

$$E_i + E_R = E_T \quad \text{———} \quad (9)$$

and $$H_i\cos\theta_i - H_R\cos\theta_R = H_T\cos\theta_T \quad \text{———} \quad (10)$$

Now as $\theta_i = \theta_R$ and $H = (E/Z) = (nE/Z_0)$ as $(\mu_r = 1)$

So equation (10) reduces to

$$n_1 E_i\cos\theta_i - n_1 E_R\cos\theta_i = n_2 E_T\cos\theta_T \quad \text{———} \quad (11)$$

Now eliminating E_T from eqn (11) with the help of (9), we get

$$(E_i - E_R)\, n_1\cos\theta_i = n_2\cos\theta_T(E_i + E_R)$$

i.e $$\left(\frac{E_R}{E_i}\right)_{\perp} = \frac{\cos\theta_i - \frac{n_2}{n_1}\cos\theta_T}{\cos\theta_i + \frac{n_2}{n_1}\cos\theta_T} \quad \text{——} (C')$$

or $$\left(\frac{E_R}{E_i}\right)_{\perp} = \frac{\cos\theta_i - \frac{\sin\theta_i}{\sin\theta_T}\cos\theta_T}{\cos\theta_i + \frac{\sin\theta_i}{\sin\theta_T}\cos\theta_T} \quad (\text{as } n_1\sin\theta_i = n_2\sin\theta_T)$$

or $$\left(\frac{E_R}{E_i}\right)_{\perp} = \frac{\sin\theta_T\cos\theta_i - \cos\theta_T\sin\theta_i}{\sin\theta_T\cos\theta_i + \cos\theta_T\sin\theta_i}$$

i.e $$\left(\frac{E_R}{E_i}\right)_{\perp} = \frac{\sin(\theta_i - \theta_T)}{\sin(\theta_i + \theta_T)} \quad \text{——} (C)$$

Similarly eliminating E_R from eqn (11) with the help of (9) we get

$$n_1 E_i \cos\theta_i - n_1(E_T - E_i)\cos\theta_i = n_2 E_2 \cos\theta_T$$

i.e $$\left(\frac{E_T}{E_i}\right)_{\perp} = \frac{2\cos\theta_i}{\cos\theta_i + \frac{n_2}{n_1}\cos\theta_T} \quad \text{——} (D')$$

or $$\left(\frac{E_T}{E_i}\right)_{\perp} = \frac{2\cos\theta_i}{\cos\theta_i + \frac{\sin\theta_i}{\sin\theta_T}\cos\theta_T}$$

or $$\left(\frac{E_T}{E_i}\right)_{\perp} = \frac{2\cos\theta_i\sin\theta_T}{\cos\theta_i\sin\theta_T + \sin\theta_i\cos\theta_T}$$

i.e $$\left(\frac{E_T}{E_i}\right)_{\perp} = \frac{2\cos\theta_i\sin\theta_T}{\sin(\theta_i + \theta_T)} \quad \text{——} (D)$$

Equation (A), (B) (C) and (D) are the desired results known as Fresnel's Law or formulae of reflection and refraction of E.M. waves

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