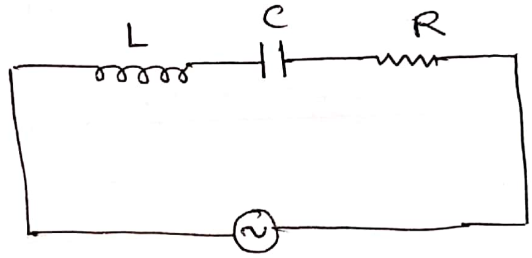
LECTURE SERIES-05

DATE 27-04-20 | B.Sc(H)-II PAPER-IV | PHYSICS ELECTRICITY | By. Dr. M.K. THAKUR A.P.S.M. College, Barauni

L-C-R Circuit (In Series): Let an alternating emf $E = E_0 \sin\omega t$ be applied to a circuit containing an inductance L, a capacitance C and a resistance R in series. Let q be the charge on the capacitor at any instant and I the current at that instant. Then the potential difference across the capacitor is q/c and the back emf due to self-inductance in L is $L\left(\frac{dI}{dt}\right)$, both being opposite to the applied emf. The effective emf in the circuit at that instant is therefore $E_0 \sin\omega t - \frac{q}{c} - L\left(\frac{dI}{dt}\right)$. But by Ohm's Law, the effective emf is equal to the p.d across the resistance, which is RI



$E = E_0 \sin\omega t$

(Fig.(1))

$$\therefore \quad E_0 \sin\omega t - \frac{q}{c} - L\left(\frac{dI}{dt}\right) = RI$$

or $$L\left(\frac{dI}{dt}\right) + RI + \frac{q}{c} = E_0 \sin\omega t$$

on differentiation, we get

$$L\left(\frac{d^2I}{dt^2}\right) + R\frac{dI}{dt} + \frac{1}{C}\cdot\frac{dq}{dt} = E_0\omega\cos\omega t$$

But $\frac{dq}{dt} = I$

$$\therefore \quad L\left(\frac{d^2I}{dt^2}\right) + R\left(\frac{dI}{dt}\right) + \frac{I}{C} = E_0\omega\cos\omega t \qquad \text{—— (1)}$$

This is differential equation of L-C-R circuit.

In the steady state the current alternates with the same frequency as the applied emf but may differ in amplitude and phase. Hence, Let the solution of equ^n (1) in the steady state be of the form

$$I = I_0 \sin(\omega t - \phi) \qquad \text{—— (2)}$$

Where I_0 and ϕ are constants to be determined. Differentiating equ^n (2) we have

$$\frac{dI}{dt} = I_0 \omega \cos(\omega t - \phi)$$

and $$\frac{d^2I}{dt^2} = -I_0 \omega^2 \sin(\omega t - \phi)$$

Substituting the values of I, $\frac{dI}{dt}$ and $\frac{d^2I}{dt^2}$ in eqn (1) we get

$$-L I_0 \omega^2 \sin(\omega t - \phi) + R I_0 \omega \cos(\omega t - \phi) + \frac{I_0}{C} \sin(\omega t - \phi)$$
$$= E_0 [\{\cos(\omega t - \phi) + \phi\}]$$
$$= E_0 [\cos(\omega t - \phi)\cos\phi - \sin(\omega t - \phi)\sin\phi)]$$

This is an identity, which must be true for all values of t. Hence the Co-efficient of $\sin(\omega t - \phi)$ and $\cos(\omega t - \phi)$ on both sides of it must be Separately equal.

$$\left(-L\omega^2 + \frac{1}{C}\right) I_0 = -E_0 \omega \sin\phi \quad \text{—— (3)}$$

and $$R\omega I_0 = \omega E_0 \cos\phi \quad \text{—— (4)}$$

Dividing eqn (3) by (4), we get

$$-\tan\phi = \frac{\omega\left(L\omega - \frac{1}{\omega C}\right)}{R\omega}$$

or $$\tan\phi = \frac{\left(L\omega - \frac{1}{\omega C}\right)}{R} = \frac{\text{Reactence}}{\text{Resistance}} \quad \text{—— (5)}$$

Squaring and additig eqn (3) and (4), we get

$$I_0^2 \left\{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right\} \omega^2 = \omega^2 E_0^2$$

$$\therefore \quad I_0 = \frac{E_0}{\sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}} \quad \text{—— (6)}$$

This is the peak (maximum) value of current, which is also known as the amplitude of current.

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Substituting for E_0 and ϕ in equ^n (2), we get

$$I = \frac{E_0}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}} \sin\left\{\omega t - \tan^{-1}\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right)\right\} \quad \text{--- (7)}$$

This equation represents the instantaneous current in the circuit. The quantity $\sqrt{\{R^2+(\omega L-\frac{1}{\omega C})^2\}}$ is called the impedance Z of the circuit.

i.e $$Z = \sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}$$

Here $X_L = \omega L$, $X_C = \frac{1}{\omega C}$ (Inductive reactance and Capacitive reactance)

and $X_L - X_C = (\omega L - \frac{1}{\omega C})$ resultant reactance.

and phase angle $$\phi = \tan^{-1}\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right) = \tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

Special cases (i) When $X_L > X_C$, ϕ is +ve, so that current lags behind the applied emf

(ii) When $X_L < X_C$, ϕ is -ve so that current leads the applied emf

(iii) When $X_L = X_C$, $\phi = 0$ and in this case the current is in phase with the amplitude emf.

When $X_L = X_C$ i.e inductive reactance X_L is equal to the capacitive reactance X_C, the impedance Z of the circuit is minimum and equal to the resistance R of the circuit, Hence the amplitude of the current $I_0 = E_0/R$ is maximum. This is the case of electrical resonance, Hence at resonance $X_L = X_C$ or $\omega L = \frac{1}{\omega C}$ or $\omega = \frac{1}{\sqrt{LC}}$

If f be the frequency of the applied emf

then $\omega = 2\pi f = \frac{1}{\sqrt{LC}}$ or $$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

Resonance frequency of the circuit