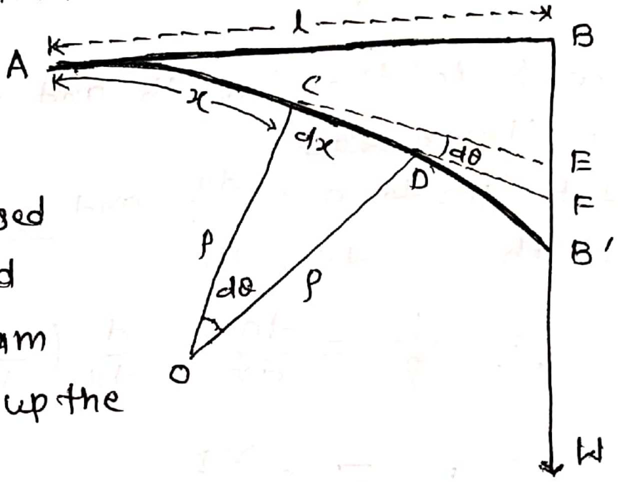
DATE 07-04-20   B.Sc(H) PART-I PAPER-I   * PHYSICS * ELASTICITY   By. Dr. M.K.THAKUR A.P.S.M. College Barauni

Depression at the loaded (Free) end of a beam

Let us consider a thin uniform and light beam AB of length l, clamped horizontally at the end A (Fig.1). When its free end B is loaded with a weight W, the end B is depressed down wards compared to fixed end A, and therefore, the beam undergoes bending and takes up the position AB'.



Consider a section of the beam at C distance x from the fixed end A of the cantilever.

Moment of the external couple acting over this section at C due to the load Mg, will be $= W \times CB'$

$$= W(l-x)$$

For equilibrium of the beam, moment of the external couple acting at the section at C should be balanced by the restoring couple (or the bending moment) YI/ρ developed as a result of the elastic reaction in the filaments i.e

$$W(l-x) = \frac{YI}{\rho} \quad \text{——} \quad (1)$$

Where ρ is the radius of curvature of the neutral axis at C.

If we consider another point D at very small distance dx from C, then we have

$$CD = \rho \cdot d\theta$$

because radius of curvature of the beam at D is practically the same as at C.

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or $\quad dx = \rho \cdot d\theta$

$\therefore \quad \rho = dx/d\theta$

Substituting this value of ρ in ①, we get

$$W(l-x) = \frac{YI \cdot d\theta}{dx}$$

or $\quad d\theta = \frac{W(l-x)}{YI} \cdot dx$ ——— ②

Let CE and DF be the tangents drawn to the beam at the points C and respectively. These tangents meet the vertical line through BB′ at E and F and they also subtend the same angle $d\theta$ between them.

Let Z be the depression of D below C (= EF)

Then $\quad Z = (l-x)\, d\theta$

$$= \frac{W(l-x)^2}{YI} \cdot dx \quad \text{(from eq}^n \text{ ②)}$$

At B (where $x = l$), the depression z of the beam is maximum. Let it be equal to δ. Hence the total depression $BB' = \delta$ of the loaded end B below the fixed end A, will be given by

$$\delta = \int_0^l \frac{W(l-x)^2}{YI}\, dx \quad \text{or} \quad \delta = \frac{Wl^3}{3YI} \text{ ——— ③}$$

This is the required expression for depression.

(i) Beam of rectangular cross-section: If the beam is of rectangular cross-section $I = bd^3/12$, Hence from eq^n ③ $\delta = 4Wl^3/Ybd^3$

(ii) Beam of circular cross-section: If the beam is of circular cross section $I = \pi r^4/4$, where r is the radius of the beam.

Hence from eq^n ③ we get

$$\delta = \frac{4Wl^3}{3Y\pi r^4}$$

——x——

Teacher's Signature: