(E.M.W)

THE POYNTING VECTOR

The most remarkable characteristic of an electromagnetic wave is that it transports energy from one region to another. We can describe the energy transfer in terms of the rate of energy flow per unit area or power per unit area by a vector $\vec{S}$, is called Poynting vector.

Let us calculate energy dW passing during time dt through a unit area held perpendicular to the direction of propagation of the wave as shown in fig.(1)

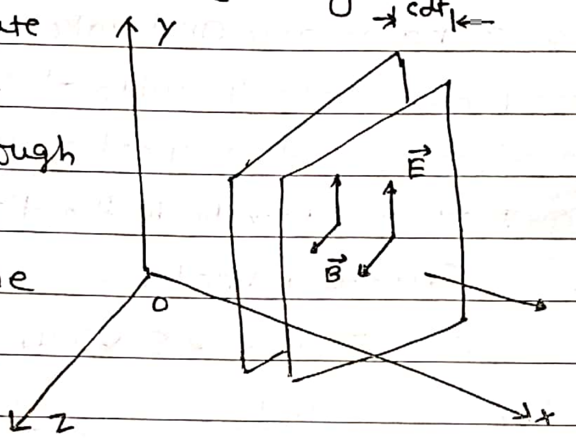

Fig.(1)

In time dt, the wave front moves a distance $dx = c\,dt$

$$\therefore \quad dW = \omega c\, dt \quad \text{——} \quad (1)$$

where ω is the energy density and is given by

$$\omega = \epsilon_0 E^2 \quad \text{——}$$

$$\therefore \quad dW = \epsilon_0 E^2 c\, dt \quad \text{——} \quad (2)$$

For an EMW $\epsilon_0 E^2 = B^2 = \mu_0 H^2$ —— (3)

which implies that the electric energy density in the E.M wave at any instant is equal to the magnetic energy density at the same point.

$$\therefore \quad \sqrt{\epsilon_0}\, E = \sqrt{\mu_0}\, H$$

$$\therefore \quad dW = \sqrt{\epsilon_0 \mu_0}\, E H c\, dt \quad \text{——} \quad (4)$$

Since

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad \text{or} \quad dW = E H\, dt \quad \text{——} \quad (5)$$

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The term EH represents the magnitude of energy flux density vecter $|\vec{S}| = EH$ —— ⑥

The cross product of the electric field vector $\vec{E}$ and magnetic field vector $\vec{H}$ is known as Poynting vecter It is denoted by $\vec{S}$ Hence poynting vecter

$$\vec{S} = \vec{E} \times \vec{H}$$

If we consider a plane polarized electro magnetic wave travelling along x-axis and take the electric vector $\vec{E}$ along z axis, then magnetic vector $\vec{H}$ will be along the y-axis, H_y becomes in an E.M wave electric and magnetic vectors are at right angles to each other and at right angle to the direction of propagation of the E.M, wave ∴ Poynting vector

$$\vec{S} = \vec{E} \times \vec{H} = \hat{k} E_z \times \hat{j} H_y = \hat{i} E_z H_y$$

Taking magnitudes only $S = E_z H_y$

Thus the poynting vector measures the flow of the electric energy per unit time per unit area held perpendicular to the direction of propagation of the electro-magnetic wave. It is also called the flux vector or power flux.

POYNTING THEOREM

As the electromagnetic wave propagates from one point to another, there is a transfer of electromagnetic energy.

Maxwell's curl equations

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{—— ①}$$

and $$\vec{\nabla} \times \vec{B} = \mu\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = \mu\left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right)$$

or $$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{—— ②}$$

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Taking the scalar product of eq^n ① with $\vec{H}$ and of eq^n ② with $\vec{E}$ we have

$$\vec{H}(\vec{\nabla}\times\vec{E}) = -\mu\frac{\partial\vec{H}}{\partial t}\cdot\vec{H} = -\frac{1}{2}\mu\frac{\partial^2\vec{H}}{\partial t} \quad \text{— ③}$$

and
$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\epsilon\frac{\partial\vec{E}}{\partial t} = \vec{E}\cdot\vec{J} + \frac{1}{2}\epsilon\frac{\partial E^2}{\partial t} \quad \text{— ④}$$

Substracting eq^n ③ from eq^n ④, we have

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) - \vec{H}\cdot(\vec{\nabla}\times\vec{E}) = \vec{E}\cdot\vec{J} + \frac{1}{2}\epsilon\frac{\partial E^2}{\partial t} + \frac{1}{2}\mu\frac{\partial H^2}{\partial t} \quad \text{— ⑤}$$

$$= \vec{E}\cdot\vec{J} + \frac{1}{2}\frac{\partial}{\partial t}[\epsilon E^2 + \mu H^2]$$

Now, $\vec{A}\cdot(\vec{\nabla}\times\vec{B}) - \vec{B}(\vec{\nabla}\times\vec{A}) = \vec{\nabla}\cdot(\vec{B}\times\vec{A}) = -\vec{\nabla}\cdot(\vec{A}\times\vec{B})$

Equation ⑤ can be put as

$$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right)$$

But $\vec{E}\times\vec{H} = \vec{S}$, the poynting vector

$$\therefore \vec{\nabla}\times\vec{S} + \frac{\partial}{\partial t}\left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right) = \vec{E}\cdot\vec{J} \quad \text{— ⑥}$$

Integrating eq^n ⑥ over a volume V bounded by the closed surface, we have

$$\oint_V \vec{\nabla}\cdot\vec{S}\,dv + \frac{\partial}{\partial t}\oint_V\left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right)dv = -\oint_V \vec{E}\cdot\vec{J}\,dv \quad \text{— ⑦}$$

According to Gauss's divergence theorem in vector

$$\oint_S \vec{S}\cdot\vec{dA} = \oint_V \vec{\nabla}\cdot\vec{S}\,dv \quad \text{— ⑦a}$$

Where $\vec{dA}$ is a small area element of the surface

Substituting the above value of $\oint_V \vec{\nabla}\cdot\vec{S}\,dv$ in eq^n ⑦, we get

$$\oint_S \vec{S}\cdot\vec{dA} + \frac{\partial}{\partial t}\oint_V\left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right)dv = -\oint_V \vec{E}\cdot\vec{J}\,dv \quad \text{— ⑧}$$

Now $\oint_S \vec{S}\cdot\vec{dA}$ = Flow of energy per unit time across the boundary of the volume V.

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$\frac{\partial}{\partial t}\oint_V \left(\frac{\epsilon E^2}{2}+\frac{\mu H^2}{2}\right) dv$ = Rate of change of total energy (Electric + magnetic) of the electromagnetic field.

$-\oint_V \vec{E}\cdot\vec{J}\, dv$ = work done by the field on the source because $\vec{E}\cdot\vec{J}$ is the energy consumed per unit volume ~~due~~

Equation (8) is the statement of Poynting theorem It gives the law of conservation of energy

In free space $\vec{J}=0$ and equ^n (8) becomes

$$\oint \vec{S}\cdot d\vec{A} = -\frac{\partial}{\partial t}\oint_V \left(\frac{\epsilon E^2}{2}+\frac{\mu H^2}{2}\right) dv \quad \text{—— (9)}$$

This equation shows that the flow of energy per unit time across the boundary of the closed volume is equal to the rate of change of total energy of the electromagnetic field.

Again substituting equ^n (9)

$$\oint \vec{S}\cdot d\vec{A} = \oint_V \vec{\nabla}\cdot\vec{S}\, dv \quad \text{from equ}^n \text{ (7a)}$$

and $$\left(\frac{\epsilon E^2}{2}+\frac{\mu H^2}{2}\right) = U_E + U_m = U \quad \text{—— (10)}$$

Where U_E is electric energy per unit volume, U_M the magnetic energy per unit volume and U the total energy per unit volume, we have $\oint_V \vec{\nabla}\cdot\vec{S}\, dv = -\oint_V \frac{\partial}{\partial t} U\, dv$

or $$\oint_V \vec{\nabla}\cdot\vec{S}\, dv + \oint_V \frac{\partial}{\partial t} U dv = 0 \quad \text{—— (11)}$$

Which gives $\vec{\nabla}\cdot\vec{S} + \frac{\partial U}{\partial t} = 0$

This is called equation of continuity

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